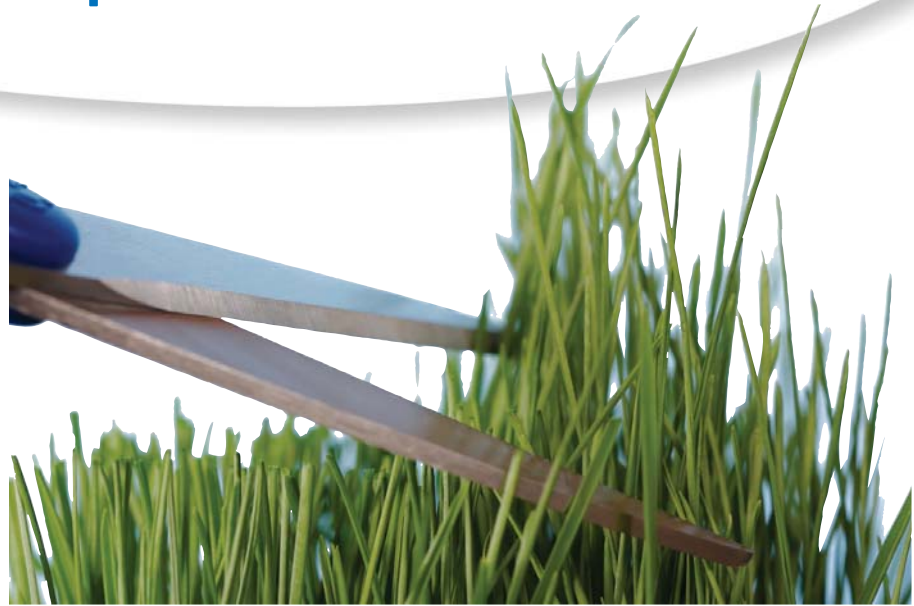


7.3

Slope-Point Form

Focus on ...

- writing the equation of a line from its slope and a point on the line
- converting equations among the various forms
- writing the equation of a line from two points on the line
- solving problems involving equations in slope-point form



You can measure the length of an arena with a ruler, use scissors to cut your grass, or loosen a screw with a paperclip. All of these are possible, but are they using the best tool for the job? In mathematics and in life, using an inappropriate tool may prevent you from completing your task or make it take longer to finish. You have explored tools for writing linear relations in two forms. This section introduces a third form, slope-point form. Each form is best suited to certain situations.

Materials

- grid paper
- ruler

Investigate Equations in Slope-Point Form

1. Square ABCD in Figure 1 is a composite of four different polygons. The lengths of the sides are shown. What is the area of square ABCD?

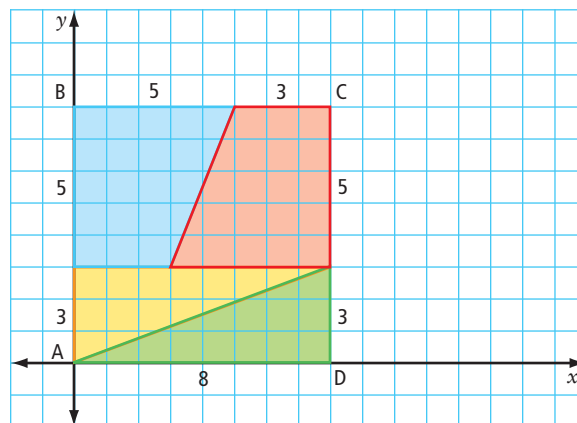


Figure 1

2. Square ABCD is reassembled to form rectangle EFGH, shown in Figure 2. What is the area of rectangle EFGH?

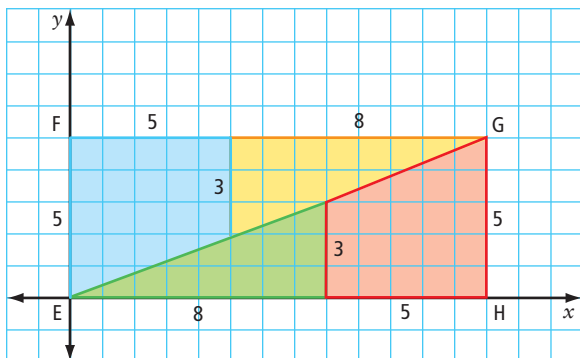


Figure 2

3. There is a discrepancy between the areas of the quadrilaterals shown in Figures 1 and 2. How is Figure 2 deceiving? Justify your answer.
4. On grid paper, draw a line that does not pass through the origin. Label points J, K, and L on the line. Determine the slope of your line.
- Determine the equation of your line using point J and the slope-intercept form, $y = mx + b$.
 - Use points K and L to determine equations of your line. Compare your equations.
5. Let $P(x_1, y_1)$ represent a point on a line. Develop an equation of the line with slope m using point P.
6. Work with a partner. Have your partner test the equation you developed using his or her line from step 4.
7. **Reflect and Respond** Describe how to determine the equation of a line using the slope and a point on the line.
8. Show how the **slope-point form** of a linear equation can be developed by using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
9. What type of line cannot be written in slope-point form? Why?
10. Is the following statement always true, sometimes true, or never true? Explain. “To determine the equation of a non-vertical line in slope-point form, you can use the coordinates of any point on the line.”

If you know one point on a line, how can you use the slope to determine a second point?

slope-point form

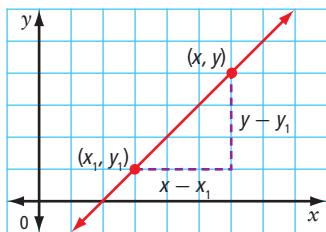
- the equation of a non-vertical line in the form $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) are the coordinates of a point on the line

Link the Ideas

The slope of a non-vertical line can be determined using $m = \frac{\Delta y}{\Delta x}$.

If (x_1, y_1) is one point on the line, then (x, y) could represent any other point on the line. Substitute the coordinates of these two points into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

The slope of the line could be written as $m = \frac{y - y_1}{x - x_1}$.



Multiplying both sides of the above equation by $(x - x_1)$ gives

$$(x - x_1)m = (x - x_1)\left(\frac{y - y_1}{x - x_1}\right)$$

$$(x - x_1)m = \cancel{(x - x_1)}^1 \left(\frac{y - y_1}{\cancel{x - x_1}_1}\right)$$

$$m(x - x_1) = y - y_1$$

This equation is called the slope-point form of a non-vertical line through point (x_1, y_1) with slope m .

The slope-point form is commonly written as $y - y_1 = m(x - x_1)$.

Example 1 Write the Equation of a Line Using a Point and the Slope

- Use slope-point form to write an equation of the line through $(-2, 5)$ with slope -3 .
- Express the equation in slope-intercept form, $y = mx + b$.
- Graph the linear relation using technology.

Solution

- Substitute -3 for m and the coordinates of the point $(-2, 5)$ for (x_1, y_1) .

$$y - y_1 = m(x - x_1)$$

$$y - (5) = -3(x - (-2))$$

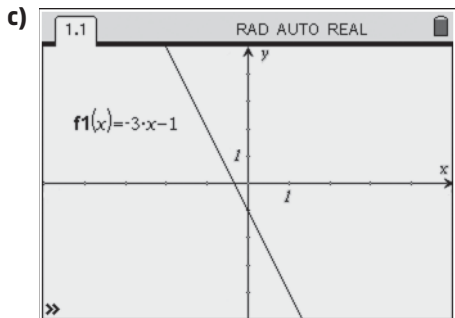
$$y - 5 = -3(x + 2)$$

The equation in slope-point form is $y - 5 = -3(x + 2)$.

- b) To express the equation in slope-intercept form, isolate y .

$$\begin{aligned}y - 5 &= -3(x + 2) \\y &= -3(x + 2) + 5 \\y &= -3x - 6 + 5 \\y &= -3x - 1\end{aligned}$$

In slope-intercept form, the equation is $y = -3x - 1$.



What strategies could you use to sketch the graph?

The equation $y = -3x - 1$ is written in the form $y = mx + b$. So, the slope is -3 . This is consistent with the value given in the question.

Your Turn

- Use slope-point form to write an equation of the line through $(3, -4)$ with slope 2. Sketch a graph of the line.
- Express the equation in slope-intercept form, $y = mx + b$. Sketch a graph of this line.
- Compare your graphs.

Example 2 Determine the Equation of a Line Using Two Points

- Use slope-point form to write an equation of the line through $(3, -4)$ and $(5, -1)$.
- Sketch a graph of the line.
- Rewrite the equation in general form, $Ax + By + C = 0$.

Solution

- a) Points on the line are given. So, you need to determine the slope. Use the two given points, $(3, -4)$ and $(5, -1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1 - (-4)}{5 - 3}$$

$$m = \frac{-1 + 4}{5 - 3}$$

$$m = \frac{3}{2}$$

In slope-point form, substitute $\frac{3}{2}$ for m and the coordinates of either point $(3, -4)$ or $(5, -1)$ for (x_1, y_1) .

Using $(3, -4)$ for (x_1, y_1) ,

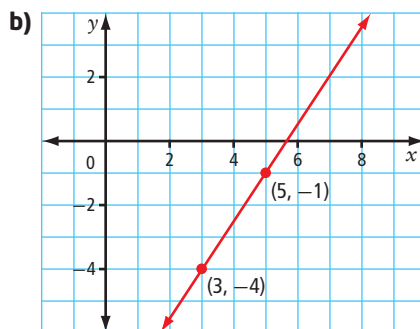
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-4) &= \frac{3}{2}(x - 3) \\ y + 4 &= \frac{3}{2}(x - 3) \end{aligned}$$

Using $(5, -1)$ for (x_1, y_1) ,

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-1) &= \frac{3}{2}(x - 5) \\ y + 1 &= \frac{3}{2}(x - 5) \end{aligned}$$

How can you verify that these equations are equivalent?

Both $y + 4 = \frac{3}{2}(x - 3)$ and $y + 1 = \frac{3}{2}(x - 5)$ are slope-point forms of the equation of the line through $(3, -4)$ and $(5, -1)$.



c) Express $y + 4 = \frac{3}{2}(x - 3)$ in general form.

$$\begin{aligned} y + 4 &= \frac{3}{2}(x - 3) \\ 2(y + 4) &= 2\left(\frac{3}{2}(x - 3)\right) \\ 2y + 8 &= 3(x - 3) \\ 2y + 8 &= 3x - 9 \\ 0 &= 3x - 9 - 2y - 8 \\ 0 &= 3x - 2y - 17 \end{aligned}$$

Express $y + 1 = \frac{3}{2}(x - 5)$ in general form.

$$\begin{aligned} y + 1 &= \frac{3}{2}(x - 5) \\ 2(y + 1) &= 2\left(\frac{3}{2}(x - 5)\right) \\ 2(y + 1) &= 3(x - 5) \\ 2y + 2 &= 3x - 15 \\ 0 &= 3x - 2y - 17 \end{aligned}$$

The equation, in general form, for the line through $(3, -4)$ and $(5, -1)$ is $3x - 2y - 17 = 0$.

Your Turn

Use slope-point form to write an equation of the line through $(-5, 2)$ and $(-2, 1)$. Explain your steps. Then, write the equation in general form, $Ax + By + C = 0$.

Example 3 Model a Real-Life Situation

Brad Zdanivsky is enthusiastic about mountain climbing. He is a quadriplegic and used custom gear as he climbed the Stawamus Chief in Squamish, BC, on July 31, 2005.

Supposed he moved at a constant rate and climbed the 660-m summit in 11 pitches (sections). Each pitch was approximately 60 m in height. At 5:45 a.m., Brad started his climb 60 m below the top of his first pitch. By 5:55 a.m., he was 40 m below the top of the first pitch.

- Write an equation that describes Brad's distance, d , in metres, below the top of the first pitch in terms of t minutes past 5:45 a.m. Express the equation in $y = mx + b$ form.
- How long did it take Brad to reach the top of the first pitch?
- In total, Brad spent 8.5 h changing ropes between pitches. How long did it take Brad to climb the Stawamus Chief?

Solution

- a) Brad was 60 m below the top of his first pitch at 0 min past 5:45 a.m. After 10 min, he was 40 m below the top of his first pitch. As coordinate pairs (t, d) , the data may be represented as $(0, 60)$ and $(10, 40)$. Use these points to determine the slope of the line.

$$m = \frac{d_2 - d_1}{t_2 - t_1}$$
$$m = \frac{60 - 40}{0 - 10}$$
$$m = -2$$

$$m = \frac{60 - 40}{0 - 10}$$
$$m = \frac{20}{-10}$$
$$m = -2$$



Brad's distance to the top of the pitch was decreasing at a rate of 2 m/min.

Substitute the slope, -2 , and the coordinates of either point $(0, 60)$ or $(10, 40)$ into the slope-point form of an equation.

$$\text{Using point } (0, 60),$$
$$d - d_1 = m(t - t_1)$$
$$d - 60 = -2(t - 0)$$
$$d - 60 = -2t$$

$$d = -2t + 60 \quad \text{How could you verify your equation?}$$

In slope-intercept form, the equation $d = -2t + 60$ represents Brad's distance below the first pitch.

WWW Web Link

To learn more about mountain climbing, go to www.mhrmath10.ca and follow the links.



- b) At the top of the first pitch, $d = 0$. Determine t .

$$d = -2t + 60$$

$$0 = -2t + 60$$

$$2t = 60$$

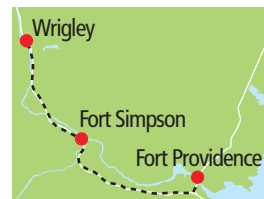
$$t = 30$$

It took Brad 30 min or 0.5 h to reach the top of the first pitch.

- c) To climb the 11 pitches, it took Brad $11(0.5 \text{ h}) = 5.5 \text{ h}$.
Adding 8.5 h to change ropes, it took Brad 14 h to climb the Stawamus Chief.

Your Turn

A family drives at a constant speed from Wrigley, NT, to visit relatives in Fort Providence, NT. When they start driving at 9:00 a.m., they are 540 km from Fort Providence. At 12:30 p.m., they reach Fort Simpson, located 225 km from Fort Providence.



- a) Write an equation that describes their distance, d , in kilometres, from Fort Providence in terms of t hours past 9:00 a.m.
b) What time will the family reach Fort Providence?

Key Ideas

- For a non-vertical line through the point (x_1, y_1) with slope m , the equation of the line can be written in slope-point form as $y - y_1 = m(x - x_1)$.

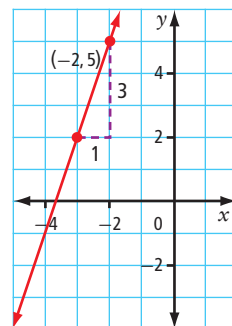
A line through $(-2, 5)$ has a slope of 3.

The slope-point form of the equation of this line is

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 3(x - (-2))$$

$$y - 5 = 3(x + 2)$$



- An equation written in slope-point form can be converted to either slope-intercept form or general form.
- Any point on a line can be used when determining the equation of the line in slope-point form.

Check Your Understanding

Practise

1. Rewrite each equation from slope-point form to slope-intercept form, $y = mx + b$, and general form, $Ax + By + C = 0$.

a) $y + 3 = x - 5$

b) $y + 4 = 2(x + 3)$

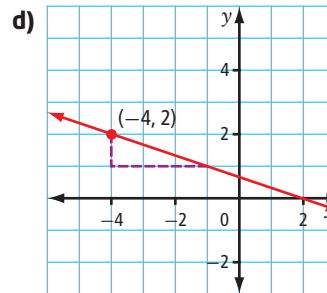
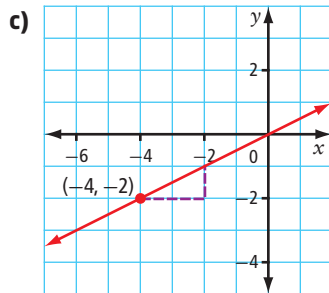
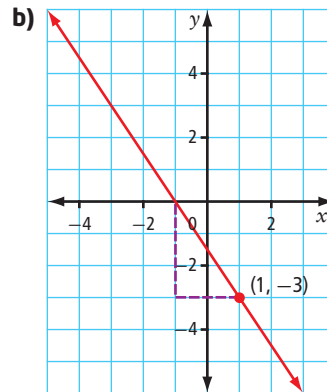
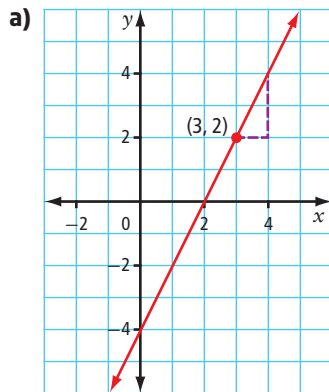
c) $y - 6 = 4(x + 1)$

d) $y + 2 = -5(x + 3)$

e) $y - 3 = -\frac{1}{2}(x + 8)$

f) $y + 9 = -\frac{2}{3}(x - 6)$

2. Write an equation in slope-point form, $y - y_1 = m(x - x_1)$, of each line passing through the given point.



3. Determine the equation of each line using slope-point form. Then, express each equation in slope-intercept form and in general form.

a) $(5, -2)$, $m = 6$

b) $(-3, -5)$, $m = -2$

c) $(-8, 3)$, $m = \frac{1}{2}$

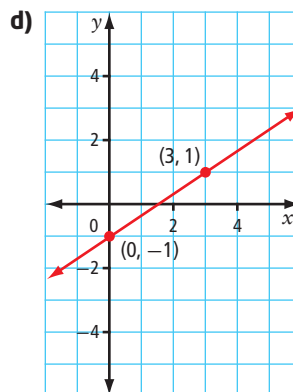
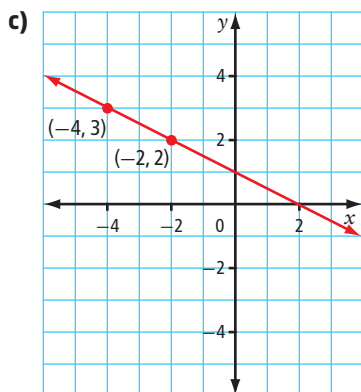
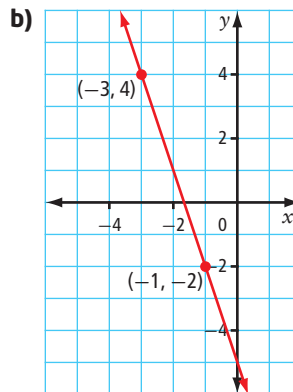
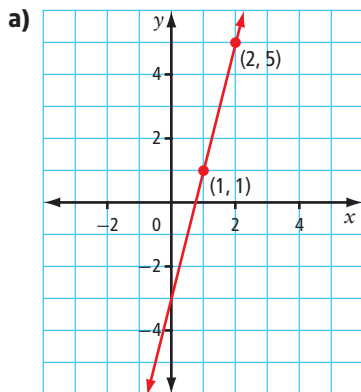
d) $(12, -6)$, $m = -\frac{2}{3}$

4. Consider the line represented by $y - 1 = \frac{2}{3}(x - 6)$.

a) Identify the slope and a point on the line.

b) Explain how you could sketch the graph of the line using the slope and a point on the line.

5. Write an equation in slope-point form, $y - y_1 = m(x - x_1)$, of the line passing through the given points.



6. Use slope-point form to write an equation of a line through each pair of points. Express each equation in the form $y = mx + b$ and in the form $Ax + By + C = 0$.

- | | |
|------------------------|------------------------|
| a) (5, 1) and (3, -7) | b) (5, -8) and (1, 4) |
| c) (4, 5) and (2, 6) | d) (8, -3) and (4, -6) |
| e) (5, -1) and (3, -4) | f) (3, 6) and (-1, 0) |

7. Terry's teacher writes the following on the board:

The four equations listed represent only two different lines.
Which equations represent the same line?

- ① $y - 2 = 3(x + 1)$
 ② $y - 10 = 3(x - 4)$
 ③ $y + 5 = 3(x + 1)$
 ④ $y - 11 = 3(x - 2)$

- a) Describe possible strategies students could use to answer the question.
 b) Which equations represent the same line? Justify your answers.

Apply

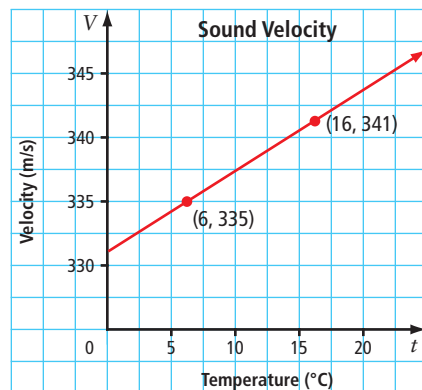
8. Identify the slope and a point on each line. Sketch a graph of each line. Use graphing technology to check your graphs.
- a) $y - 3 = 2(x - 1)$ b) $y + 2 = -3x$
c) $y - 4 = \frac{1}{2}(x + 1)$ d) $y + 6 = -\frac{4}{5}(x - 2)$
9. Consider the line passing through the points $(-4, 2)$ and $(-2, 6)$.
- a) Work with a partner to develop at least two different strategies for determining the y -intercept of the line.
b) What is the y -intercept of the line?
10. A line passes through $(0, 1)$ and $(3, 7)$.
- a) Using only slope-intercept form, $y = mx + b$, write the equation of this line.
b) Determine the equation of the line using only slope-point form.
c) Compare the two equations graphically.
11. Write the equation of each line using slope-point form. Then, convert to slope-intercept form.
- a) slope of 0 and through $(4, -5)$
b) same slope as $3x + y = 5$ and through $(-2, 4)$
c) same slope as the line $x - 2y + 6 = 0$ and the same x -intercept as the line $3x - 2y = 24$
d) same y -intercept as $x + 4y = 8$ and through $(3, -4)$
12. What is the equation of each line in slope-point form? Convert each equation to general form.
- a) slope of 3 and x -intercept of 4
b) same slope as $y = -4x + 5$ and through $(2, -1)$
c) same x -intercept as the line $3x + y = 12$ and through $(0, 2)$
d) x -intercept of 2 and y -intercept of -6
13. An “iron horse” pumpjack starts to pump crude oil into a tank at a constant rate of $1.2 \text{ m}^3/\text{h}$. After 24 h, the tank contains 29 m^3 of oil.
- a) Write an equation that describes the volume, V , in cubic metres, of oil in the tank after t hours.
b) The tank can hold a maximum of 155 m^3 of oil. How long will it take to fill the tank?
c) Was the tank empty before it started filling? Explain.



Did You Know?

Oil pumpjacks are common in western Canada. They are a traditional method of oil recovery. The surface deposits of the Athabasca Oil Sands in present day northern Alberta were once used by the Cree and Dene peoples to waterproof their canoes and other items.

14. The graph shows the linear relationship between the velocity of sound, V , in metres per second, and the temperature, t , in degrees Celsius, of dry air. At $6\text{ }^{\circ}\text{C}$, the velocity of sound is 335 m/s . At $16\text{ }^{\circ}\text{C}$, it is 341 m/s .



- What is the slope of the line?
 - What rate of change does the slope represent?
 - What is the equation of the line?
 - Determine the velocity of sound at $35\text{ }^{\circ}\text{C}$.
 - What is the air temperature when the velocity of sound is 348 m/s ?
15. What is the y -intercept of a line with a slope of $\frac{1}{2}$ and an x -intercept of 4?
16. Determine the x -intercept of a line through $(3, 4)$ having a y -intercept of 2.
17. Suppose Canada's population has grown steadily since 2000. In 2001, the population was 30.0 million. In 2009, it was 33.7 million.
- Let t represent the number of years since 2000. Let p represent the population of Canada in millions. Write the coordinates of two points in the form (t, p) .
 - Determine the slope of the line through the points.
 - What rate does the slope represent?
 - Write an equation to represent population growth in Canada since 2000.
 - Predict Canada's population in 2017.

WWW Web Link

To learn more about the components of population growth in parts of Canada, go to www.mhrmath10.ca and follow the links.



18. Suppose your friend's dinner tonight consists of one steak and mini potatoes. The steak has approximately 30 g of protein. The nutrition facts label shows the number of grams of protein per number of mini potatoes.

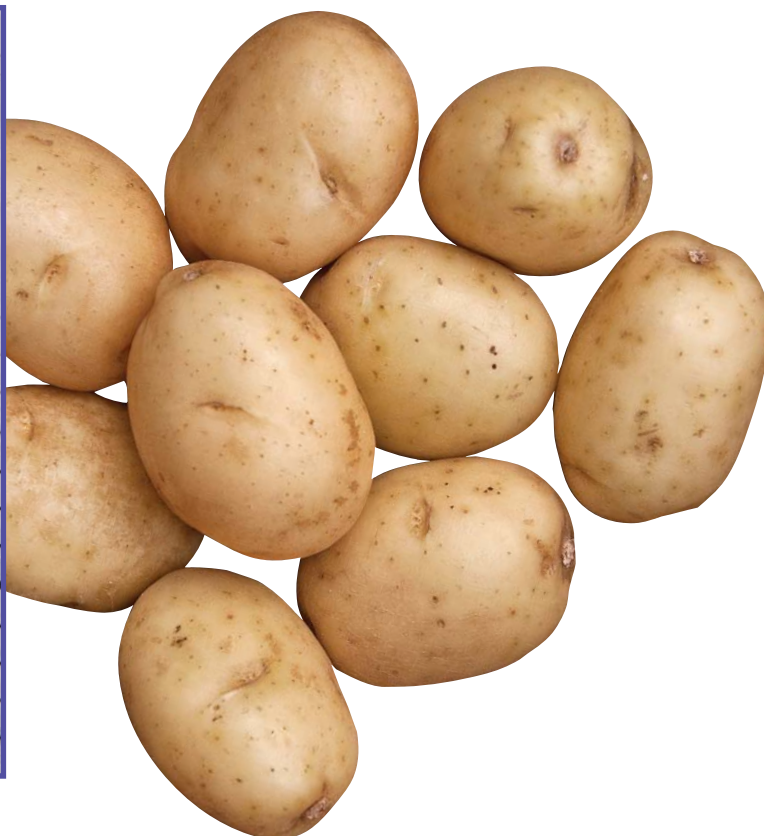
- Write an equation relating the protein, p , in the meal to the number of potatoes, n , eaten. Use the data in the nutrition facts label.
- What is the slope of the line? What does the slope represent?
- What is the p -intercept of the line? Why is the p -intercept not zero?
- Suggest a reasonable domain and range for the graph.

Did You Know?

In Canada, potato production is a multi-million-dollar industry. Manitoba produces the second greatest amount of potatoes in Canada.

Nutrition Facts / Valeur nutritive		Yellow Potatoes Pommes de terre jaunes (110 g)
Serving Size 5 potatoes Portions 5 pomme de terre		
	Amount Teneur	% DV* % VQ*
Calories / Calories	80	
Fat / Lipides	0 g	0 %
Saturated / saturés + Trans / trans	0 g 0 g	0 %
Cholesterol / Cholestérol	0 mg	0 %
Sodium / Sodium	2 mg	0 %
Potassium / Potassium	480 mg	14 %
Carbohydrate / Glucides	18 g	6 %
Fibre / Fibres	2 g	8 %
Sugars / Sucres	1 g	
Protein / Protéines	3 g	
Vitamin A / Vitamine A		0 %
Vitamin C / Vitamine C		4 %
Calcium / Calcium		0 %
Iron / Fer		6 %

* DV = Daily Value / VQ = valeur quotidienne



Extend

- Write the equation of a line with an x -intercept of n and a slope of m .
- A line passes through the point of intersection of the lines $y = -\frac{1}{2}x - 6$ and $y = 2x + 4$. Determine the equation of the line if it has a slope of $\frac{1}{2}$.

Create Connections

21. How can you develop the slope-intercept form, $y = mx + b$, by substituting a point into $y - y_1 = m(x - x_1)$?
22. To determine the equation of a line in slope-point form, you need to know two pieces of information about the line. List three sets of information that would allow you to determine the equation of a line.
23. To solve a particular problem you may want to write a linear equation in one of the three forms. You may wish to use slope-intercept form, $y = mx + b$; general form, $Ax + By + C = 0$; or slope-point form, $y - y_1 = m(x - x_1)$. Create a visual that helps you decide which form you should start with.
24. **MINI LAB** (Unit Project) Paleontologists can predict the anatomy of humans and animals based on skeletal remains.

Materials

- SI measuring tape
- grid paper
- ruler



- Step 1** Work with a partner of the same gender as yourself. Measure and record the length of each other's humerus bone. It runs from the shoulder to the elbow. Measure and record each other's height without shoes.
- Step 2** Collect and share your data with other students of the same gender. Record all data. Use grid paper to plot the data as coordinate pairs. Label the axes and scale used.
- Step 3** Draw a straight line that represents the data. What is the equation of this line?
- Step 4** Measure the humerus bone of a teacher of the same gender as you. Use your equation to predict the height of the teacher. Compare the teacher's actual height with your predicted height.